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Exercise 1

x = # of cars produced by Mr. Grecchi in the 5 years

$\delta_1 = 1$ invest in advertisement

$\delta_2 = 1$ stop outsourcing

$\delta_3 = 1 \leftrightarrow x > 0.6 \cdot 5000 \cdot 5$

$$\boxed{x \leq 0.5 \cdot 5000 \cdot 5 + 0.15 \cdot 5000 \cdot 5\delta_1}$$

20% cost reduction

$$\max_{x,\delta_1,\delta_2,\delta_3} (20000 - 10000)x + 2000x\delta_2 - 30 \cdot 10^6 \delta_1 - 20 \cdot 10^6 \delta_2 - 2000x\delta_3$$

assume 2000€ fee per each car if x exceeds 60% production

Other constraints

$$10x + 1x\delta_2 + 5000\delta_2 \leq 40000 \cdot 5$$

$$x \geq 0,\ \delta_1, \delta_2, \delta_3 \in \{0, 1\}$$

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There are two issues to be solved: the bilinearities $x\delta_2$ and $x\delta_3$ and the translation of the logical constraint $\delta_3 = 1 \iff \dots$

According to the text, the bilinearities can be tackled as follows

Mr Grecchi won't exceed 65% of production according to the possible measures

$m = 0 \qquad M = 16250 = 0.65 \cdot 5000 \cdot 5$

$y_1 = x\delta_2 \rightarrow$ replaced by $\begin{cases} y_1 \le 16250\delta_2 \\ y_1 \ge 0 \\ y_1 \le x \\ y_1 \ge x - 16250(1-\delta_2) \end{cases}$

$y_2 = x\delta_3 \rightarrow \begin{cases} y_2 \le 16250\delta_3 \\ y_2 \ge 0 \\ y_2 \le x \\ y_2 \ge x - 16250(1-\delta_3) \end{cases}$

$\delta_3 = 1 \iff x > 15000 \qquad \overset{L}{-15000} \le x - 15000 \le \overset{U}{1250}$

↓ translated as

$\delta_3 = 1 \rightarrow x \ge 15000 + \varepsilon \mid \rightarrow x - 15000 \ge \varepsilon + (L - \varepsilon)(1-\delta_3)$

$\delta_3 = 0 \rightarrow x \le 15000 \rightarrow x - 15000 \le U\delta_3$

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The problem can now be rewritten as follows

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$$\max_{\substack{x,\delta_1,\delta_2,\delta_3 \\ y_1,y_2}} 10000x + 2000y_1 - 30\cdot10^6\delta_1 - 20\cdot10^6\delta_2 - 2000y_2$$

$$x \leq 12500 + 3750\delta_1$$

$$y_1 \leq 16250\delta_2$$

$$y_1 \geq 0$$

$$y_1 \leq x$$

$$y_1 \geq x - 16250(1-\delta_2)$$

$$y_2 \leq 16250\delta_3$$

$$y_2 \geq 0$$

$$y_2 \leq x$$

$$y_2 \geq x - 16250(1-\delta_3)$$

a) $x - 15000 \geq \varepsilon + (-15000-\varepsilon)(1-\delta_3)$

$$x - 15000 \leq 1250\delta_3$$

$$10x + 1y_1 + 5000\delta_2 \leq 200000$$

a) $x - \cancel{15000} \geq \cancel{\varepsilon} - \cancel{15000} - \cancel{\varepsilon} + 15000\delta_3 + \varepsilon\delta_3$

$(15000+\varepsilon)\delta_3 - x \leq 0$

$\max\ 10000x + 2000y_1 - 30\cdot 10^6\delta_1 - 20\cdot 10^6\delta_2 - 2000y_2$ (4)

$x - 3750\delta_1 \le 12500$

$y_1 - 16250\delta_2 \le 0$

$-y_1 \le 0$ A

$y_1 - \cancel{x} \le 0$

$x + 16250\delta_2 - y_1 \le 16250$

$y_2 - 16250\delta_3 \le 0$

$-y_2 \le 0$ B

$y_2 - x \le 0$

$x + 16250\delta_3 - y_2 \le 16250$

$-x + (15000 + \varepsilon)\delta_3 \le 0$

$x - 1250\delta_3 \le 15000$

$10x + y_1 + 5000\delta_2 \le 200000$

$x, y_1, y_2 \ge 0$

$\delta_1, \delta_2, \delta_3 \in \{0, 1\}$

Constraints A and B are redundant (positivity constraints) and can be dropped. We therefore need 10 slack variables $s_1, \dots s_{10}$

Let $\bar{x}^T = [x\ y_1\ y_2\ \delta_1\ \delta_2\ \delta_3\ s_1 \cdots s_{10}]$

Then $\bar{c}^T = [10^4\ \ 2\cdot 10^3\ \ -2\cdot 10^3\ \ -3\cdot 10^7\ \ -2\cdot 10^7\ \ 0$

$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccccc} x & y_1 & y_2 & \delta_1 & \delta_2 & \delta_3 & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} \end{array} \\
A = & \begin{bmatrix}
1 & 0 & 0 & -3750 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -16250 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 16250 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -16250 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 16250 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 15000+\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -1250 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
10 & 1 & 0 & 0 & 5000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}$$

$$b^T = [12500 \;\; 0 \;\; 0 \;\; 16250 \;\; 0 \;\; 0 \;\; 16250 \;\; 0 \;\; 15000 \;\; 200000]$$

Finally

$$\max_{\bar{x}} \; c^T \bar{x}$$

$$A\bar{x} = b$$

$$\bar{x} \geq 0$$

$$\delta_1, \delta_2, \delta_3 \in \{0,1\} \leftarrow \begin{array}{l} \delta_1 = \bar{x}(4) \\ \delta_2 = \bar{x}(5) \\ \delta_3 = \bar{x}(6) \end{array} \rightarrow \begin{array}{l} \bar{x}(4), \bar{x}(5), \bar{x}(6) \\ \in \{0,1\} \end{array}$$

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